# Chapter 10
# Multi-Dimensional Fourier Imaging and Slice excitation

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- **Previous sessions:**
  - 1D Fourier transform(Chap.9)
  - Spatial encoding (Chap.9)

- **Today's content**
  - 2D/3D Fourier transform
  - Slice selective excitation
  - 2D/3D MRI, phase encoding

# Previous question: why the k-space symmetry?

- By definition, k-space is conjugate (Hermitian) symmetric to k=0, i.e.

  real (s(k)) = real(s(-k))

  imag(s(k)) = -imag(s(-k))

- This symmetry is fully valid only when signal is real, i.e. no phase term for all spins

$$\text{s(k)}= \int dz\rho(z)e^{-i\phi(r,t)}e^{-i2\pi kz}$$

$$\text{s}(k') = s(k + \frac{\phi(r,t)}{2\pi z})$$

- Using the full k-space can improve SNR over partial k-space

- Since the k-space symmetry is not practically feasible, thus one needs to acquire both + and – part of the k-space, though not necessary from –$k_{max}$ to +$k_{max}$ (e.g. partial Fourier)

# 3D imaging representation

1D: $s(k)=\int dz\rho(z)e^{-i2\pi zk}$

$$\Downarrow \quad z \rightarrow \vec{r} = (x, y, z)$$

3D: $s(\vec{k})=\int d^3\vec{r}\rho(\vec{r})e^{-i2\pi\vec{r}\cdot\vec{k}}$

1) All dimensions are equal
2) k along each direction are determined by the gradient moment in that direction

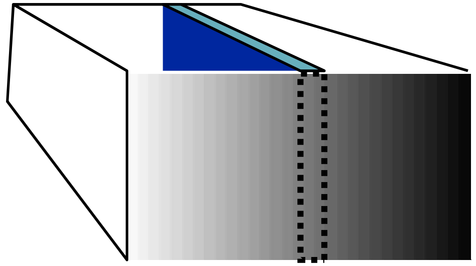
$$s(k_x, k_y, k_z)=\iiint dxdydz\, \rho(x, y, z)e^{-i2\pi(k_x x+k_y y+k_z z)}$$

# Slice Selective Excitation

- For 2D imaging, selectively excite a thin slice of the object is required
- Realized by combination of gradient fields and rf pulse with proper bandwidth and frequency
- Ideally with boxcar excitation, i.e. slice profile is rectangular

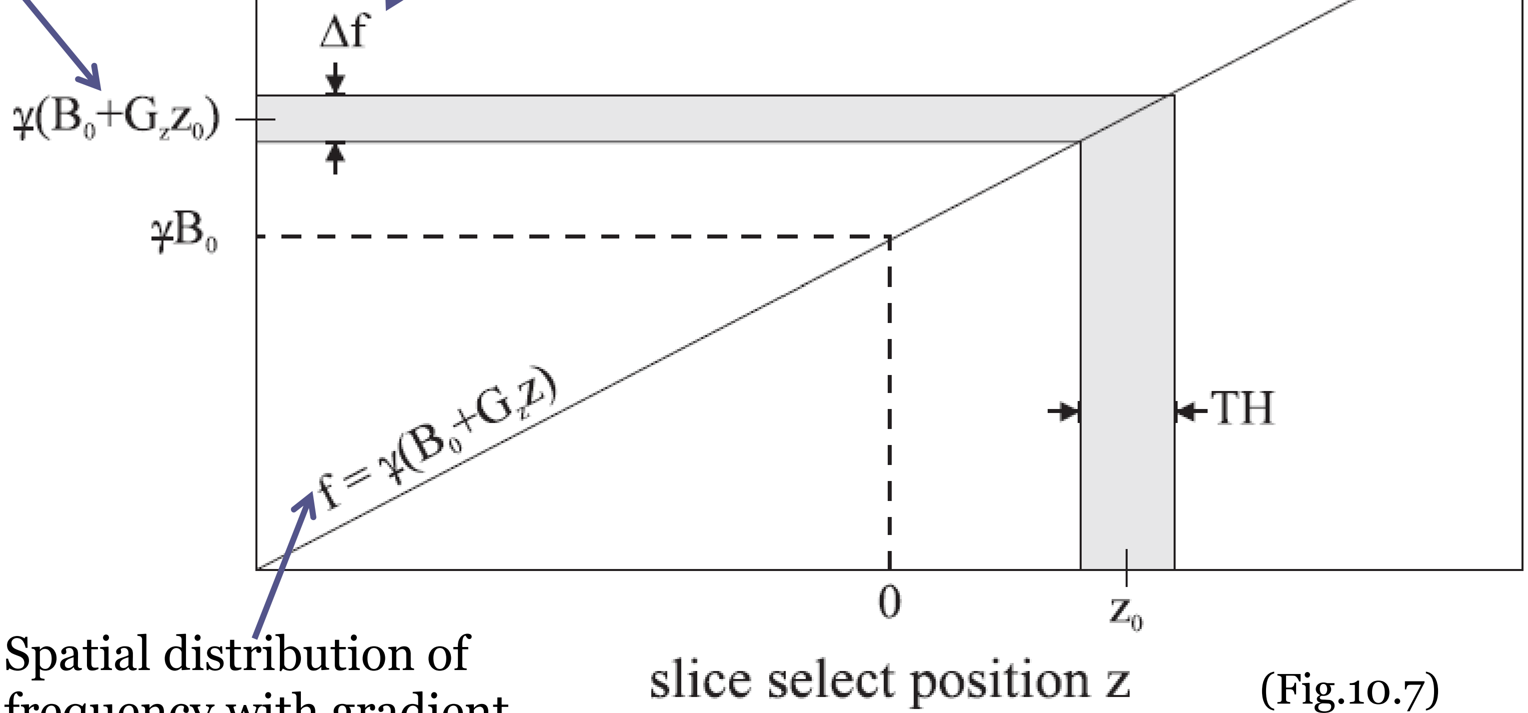
# Slice selection

Note:
$G_{ss}$ can be along arbitrary directions



RF pulse bandwidth

RF center frequency




Spatial distribution of frequency with gradient

(Fig.10.7)

# Slice profile

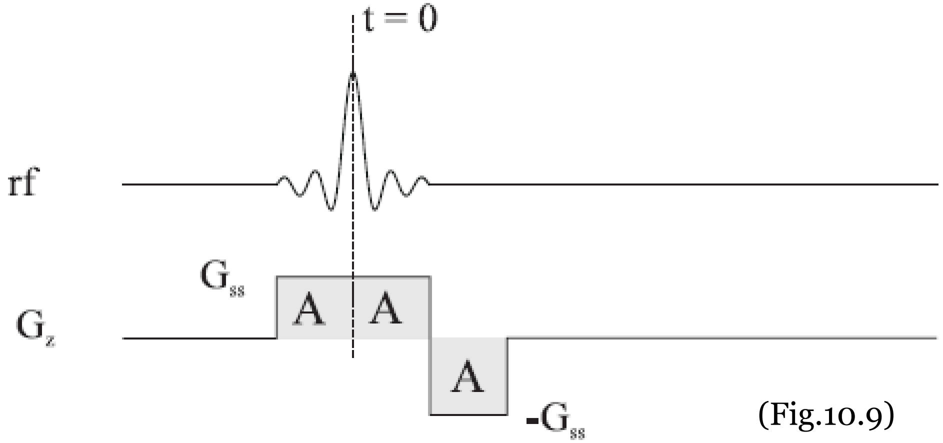
- With a linear gradient, the slice profile resembles the FT of the RF pulse
- To obtain a boxcar slice profile, one needs a infinitely long sinc-shaped RF pulse
- The bandwidth of the sinc pulse can be estimated by the number of its zero crossings

# Slice selection gradient refocusing

- Why the refocusing gradient?

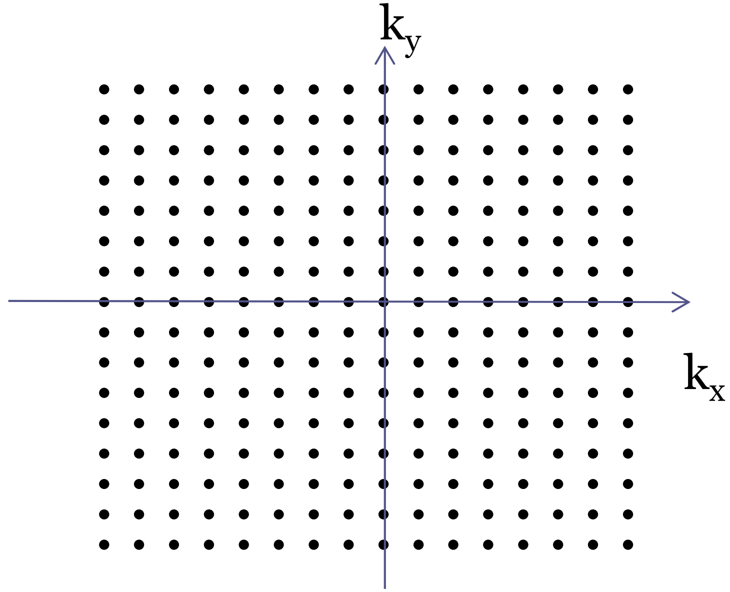
  Assumption (Chap. 16.4, low flip angle excitation):
  Spins are considered instantaneously excited at the center of RF pulse, so that spins are dephased by part of $G_{ss}$

- When and how to do the refocusing?
  - Rule of Thumb
    - Zero sum of gradient moments
    - Do it before data readout
  - Depend on the RF pulse



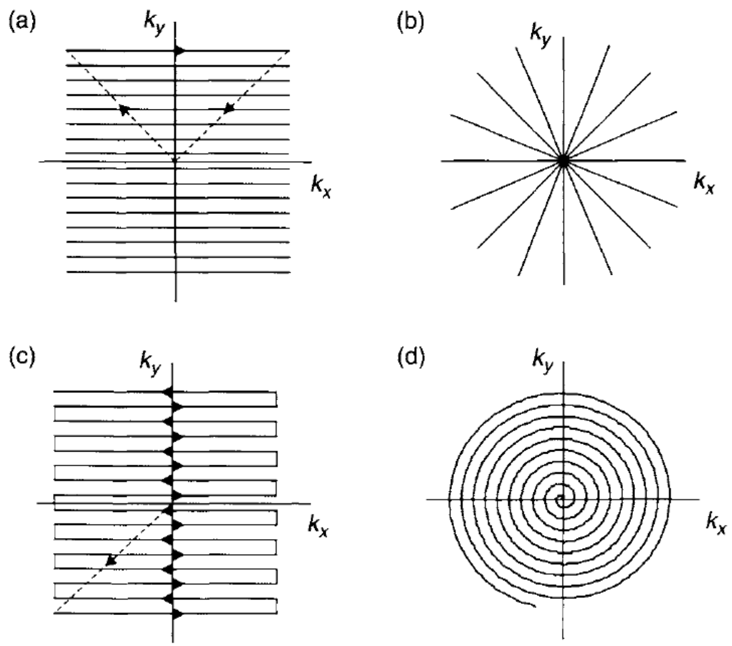

(Fig.10.9)

# 2D k-space




- The goal:
  Collect every point in the whole k-space

- The means
  Use gradients to traverse the k-space
  $$k_i(t) = \gamma \int G_i(t')dt' \text{, i = x, y, z}$$




(Bernstein et al, Handbook of MRI Pulse Sequences, Fig.11.6)

# 2D GE Sequence

- In Cartesian k-space:

$k_x$: Frequency encoding, or Readout (RO)

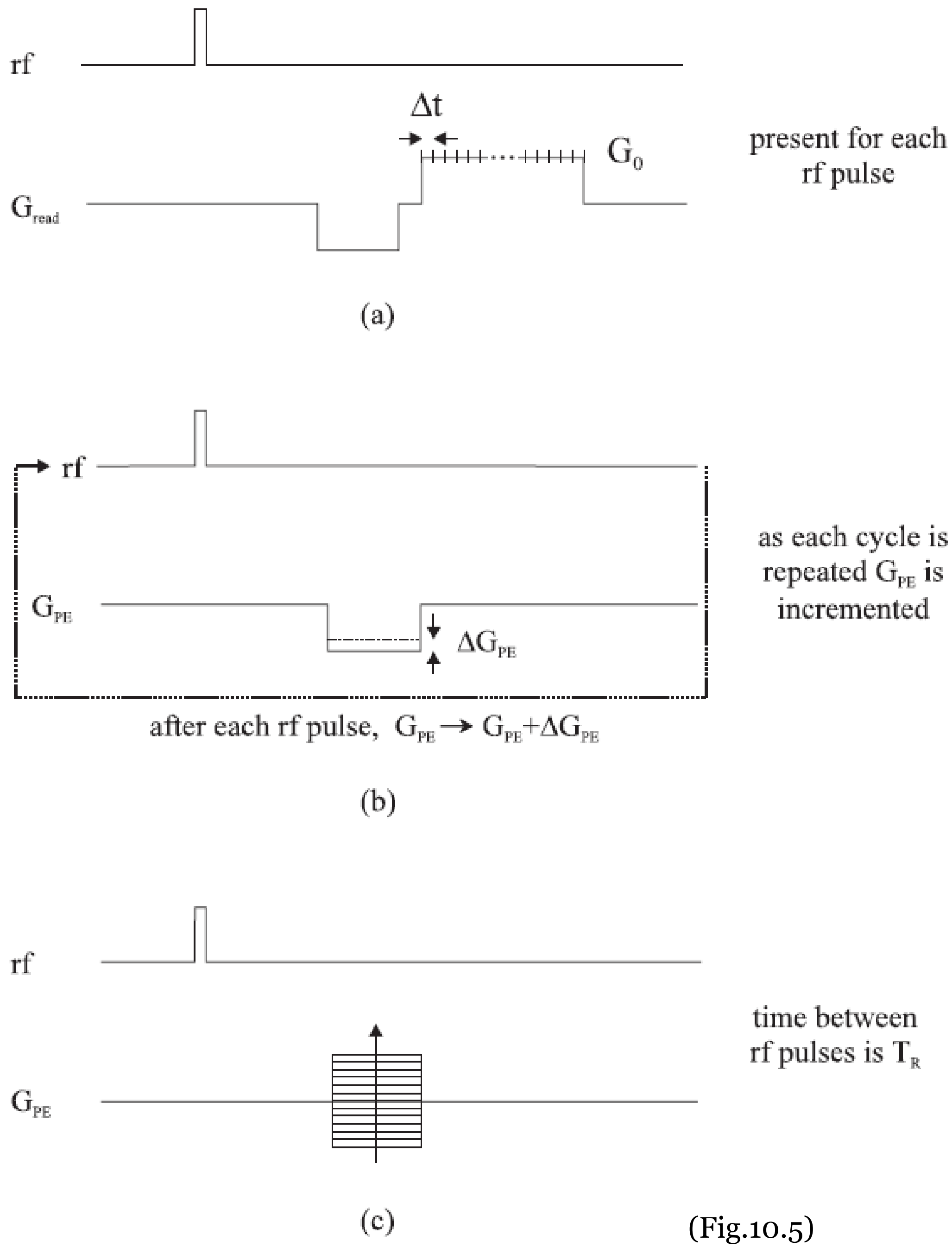
$$\Delta k_x = \gamma G_{RO} \Delta t$$

$$k_x = \gamma G_{RO} \int dt$$

$k_y$: Phase encoding, or PE

$$\Delta k_y = \gamma \Delta G_{PE} \tau$$

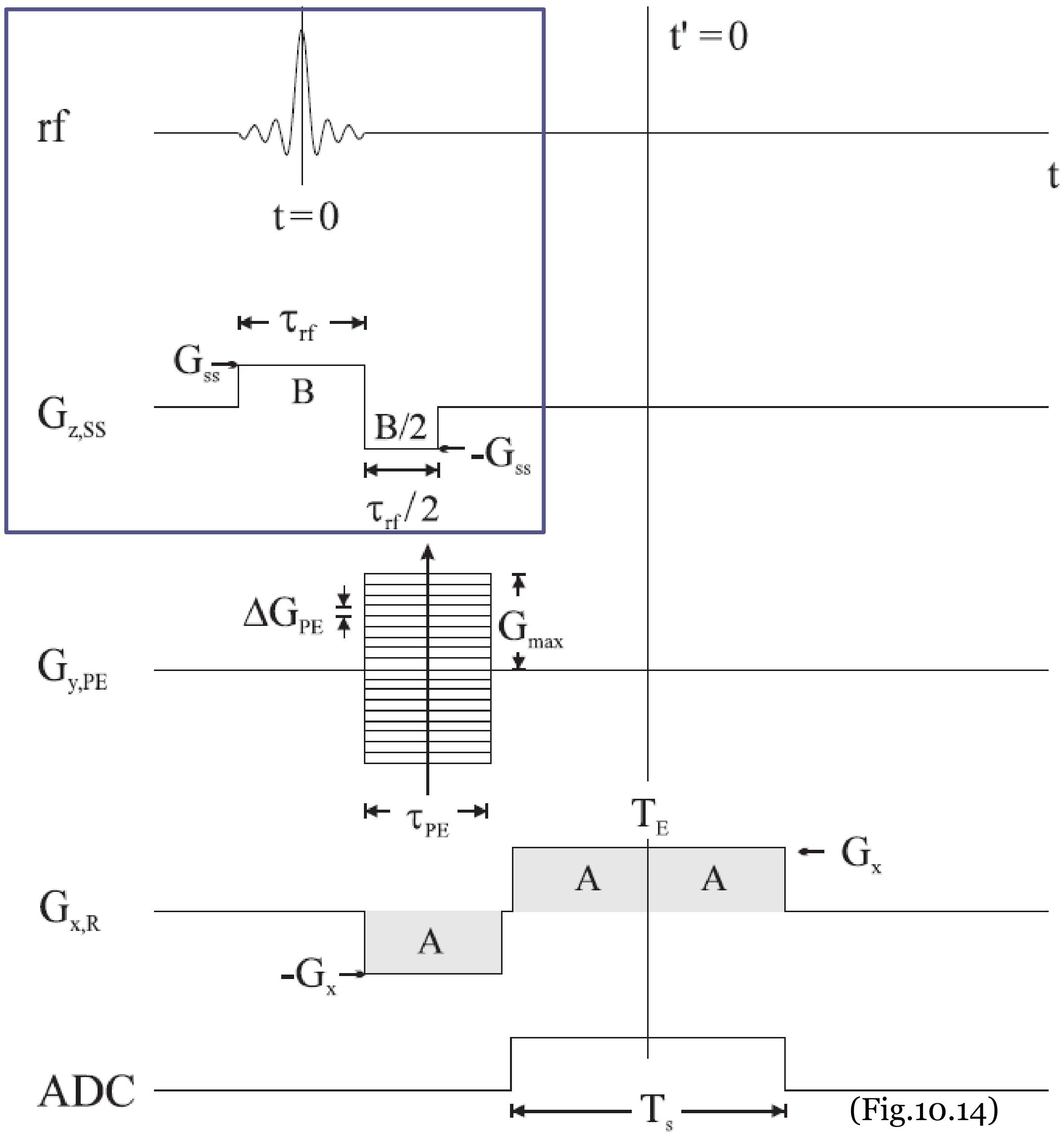
$$k_y = \gamma G_{PE} \tau$$




(Fig.10.5)

# 2D GE Diagram

- Slice selection and rephasing
- Phase encoding, one step in each TR
- Frequency encoding to readout one $k_x$ line at a time
- Pre-phasing gradient to create the 'echo'




(Fig.10.14)

# Phase encoding order (Cartesian)

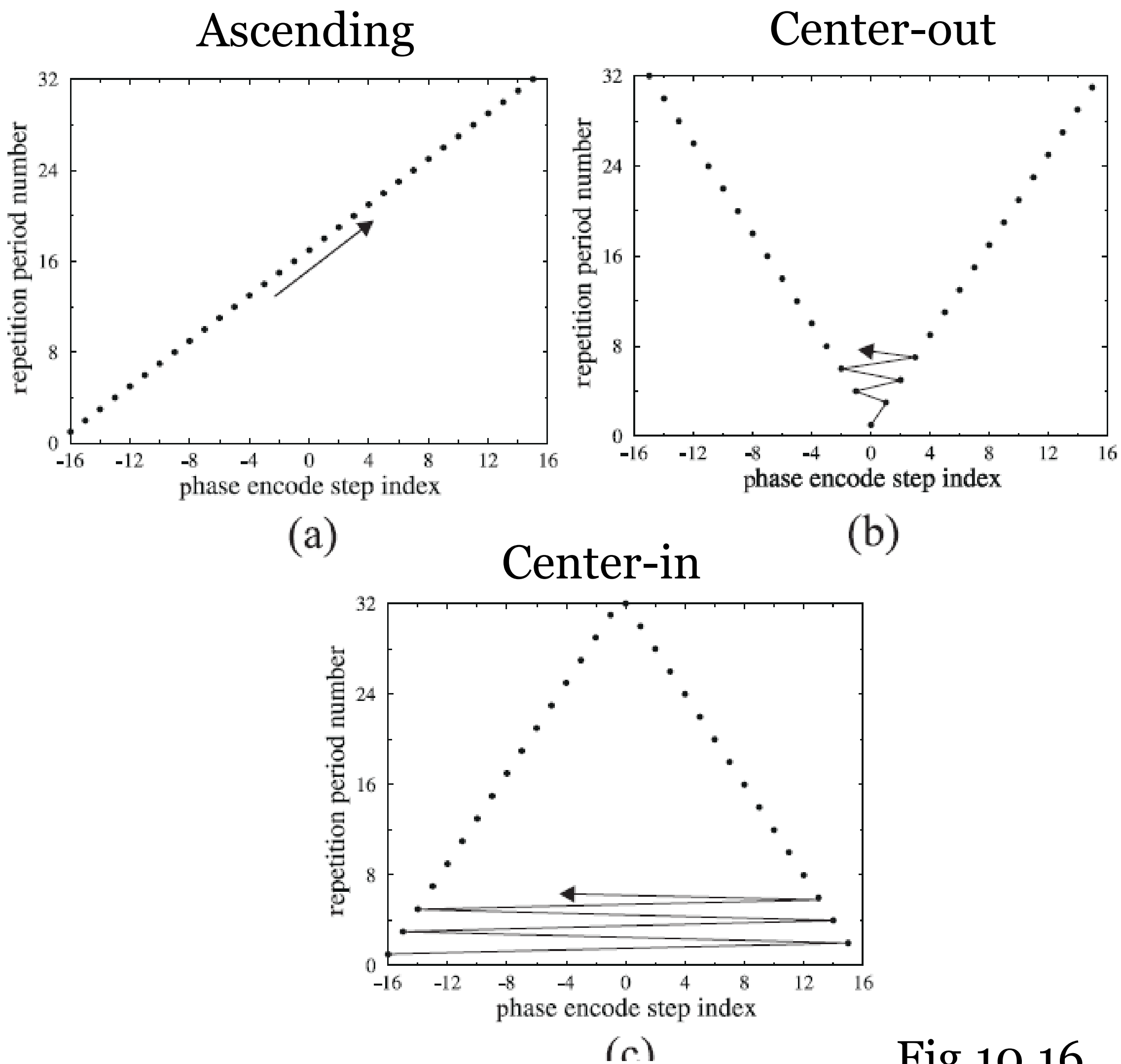
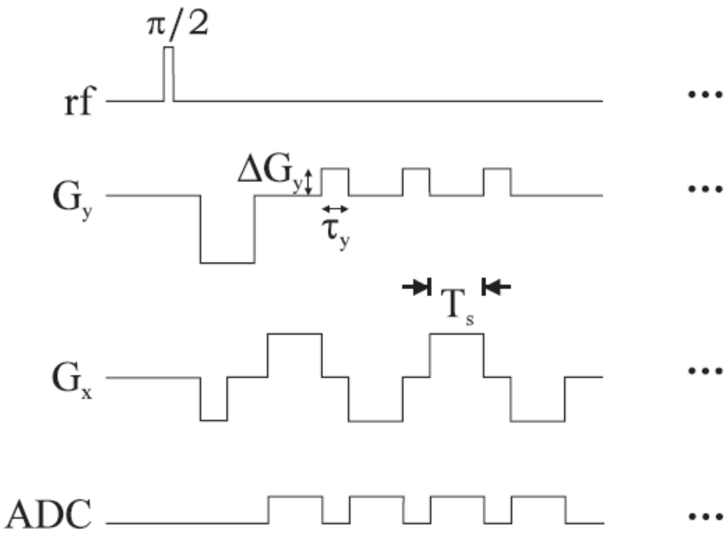



Fig.10.16

# 2D Echo Planar Imaging (EPI)

- Slice select the same as GE
- Accumulative PE effects for different echoes (blips)
- Alternating polarity for RO gradients
- Sequence characteristics:
  - Single shot whole k-space acquisition, <30ms/image
  - Sequential k-space filling
  - Heavy T2* weighting, low resolution, high RO bandwidth



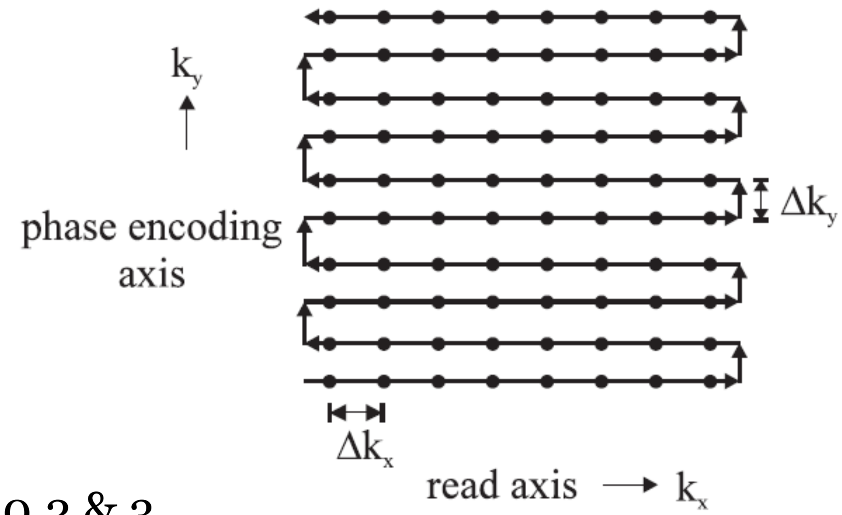




Fig. 10.2 & 3

# Phase Encoding vs. Frequency Encoding

- Same in terms of
  - k definition
  - Phase effect to the spins

- Different in terms of
  - Spatial/k direction
  - Acquisition: Discrete vs. continuous
  - Effective acquisition bandwidth: 1/TR vs. $1/T_s$
  - General contribution to scan time (depend on seq types)
    Single echo seq:
    RO: min TR/TE PE: total scan time
    Single shot seq:
    RO+PE: min TR
    Multi-shot + multi-echo (segmented) seq:
    Complecated

# Phase effects superposition of gradients

- Gradient-only effects are independent and linearly addable in any direction (e.g. PE/RO prephase/SS dephase)
- Such effects are only reflected on the transversal magnetizations
- With other components (e.g. RF pulse), this may not be true
- Make use according to needs (especially when with RF pulse, ADC and directional requirements)

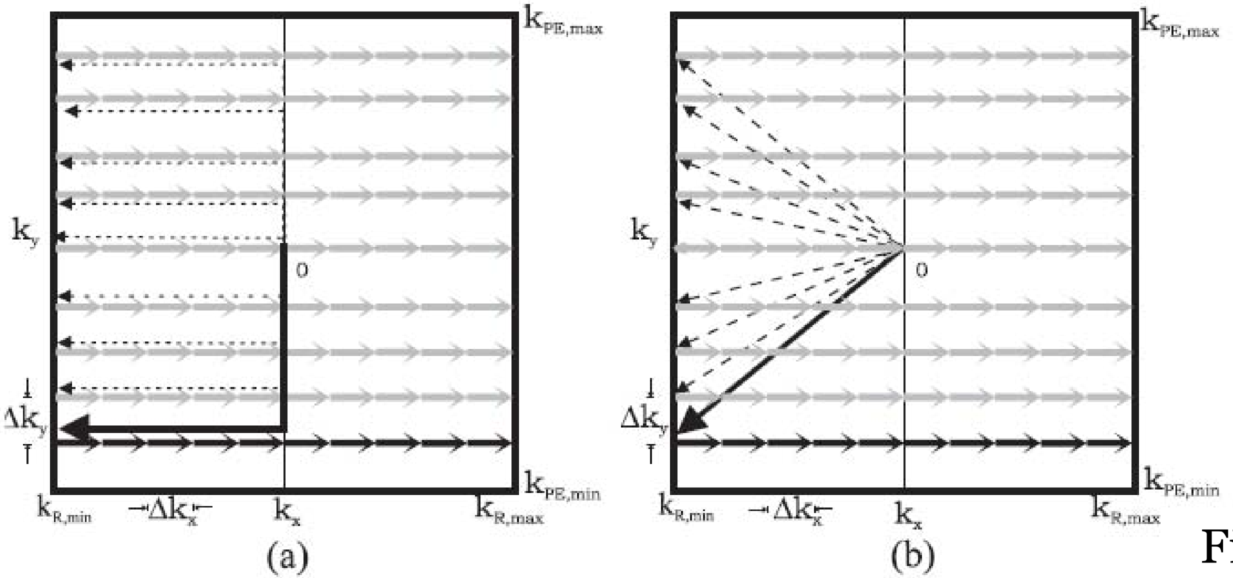



Fig.10.15

# Multi-slice 2D imaging

- Same $G_{ss}$ but varying RF center frequency
- Slice gap is usually needed due to imperfect RF frequency profiles
- What is the TR here? (using the dead time)

(Hint: TR can be defined as the interval between Readouts for the same slice)

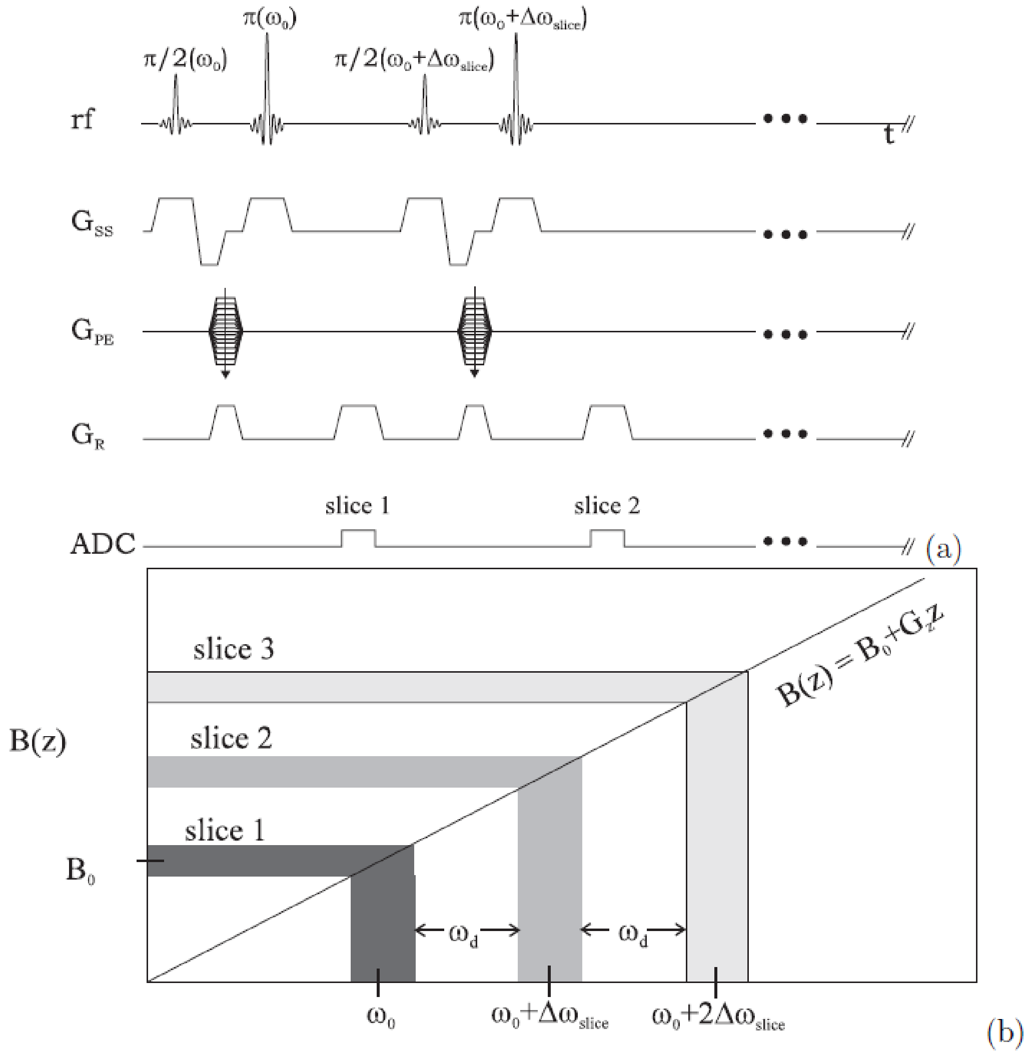



Fig. 10.19

# 3D Volumetric Imaging

- Additional PE along slice selection
- Volumetric excitation (slab instead of slice)
- 3D iFFT
- Pros (vs. multi-slice 2D):
  - High resolution on SS dimension
  - High SNR
- Cons (vs. multi-slice 2D):
  - Longer scan time
  - Min slice# per slab for FFT
  - Sensitive to motion

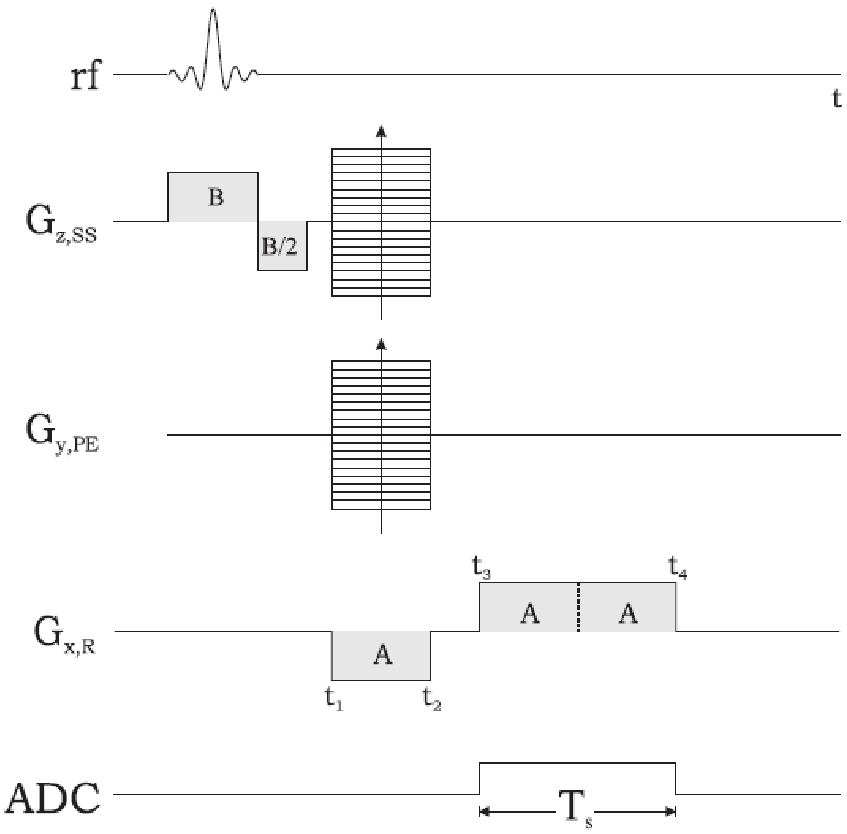
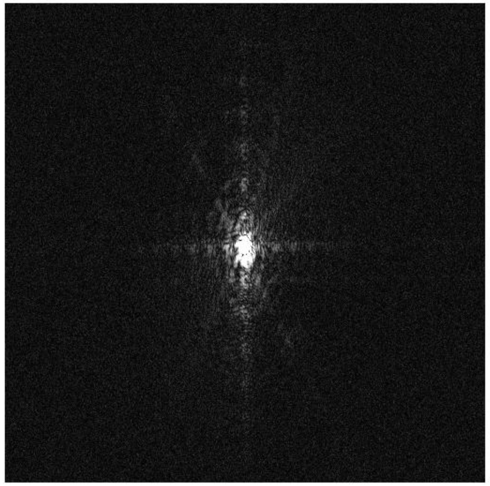



Fig.10.18

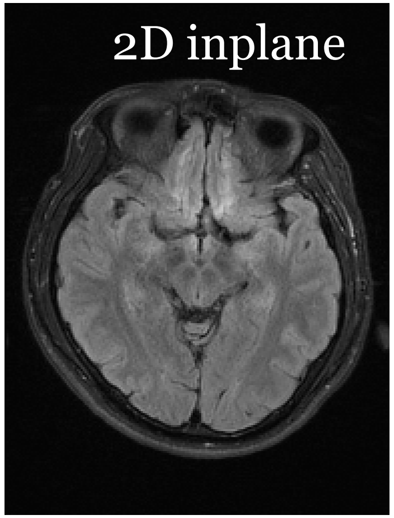
# Multi-slice 2D vs. 3D

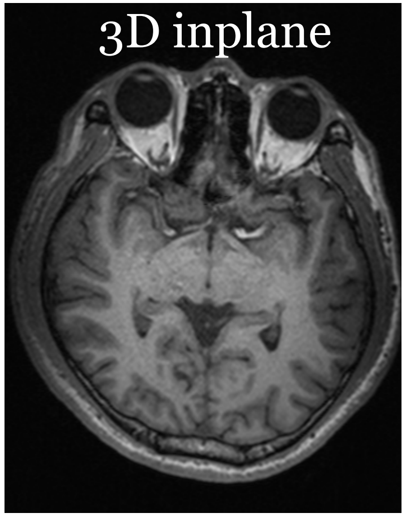
- Excitation:
  Slice selective vs. slab selective
- Within TR:
  Multiple excitation vs. single excitation
- Reconstruction I:
  No recon over SS vs. FFT along SS (or partition)
- Reconstruction II:
  Capable of single slice vs. not
- Slice resolution:
  Low (thick slice, >2mm) vs. High (thin slice, down to 0.5mm)
- SNR:
  $N_z$ times lower noise in 3D than 2D (Chap. 15)

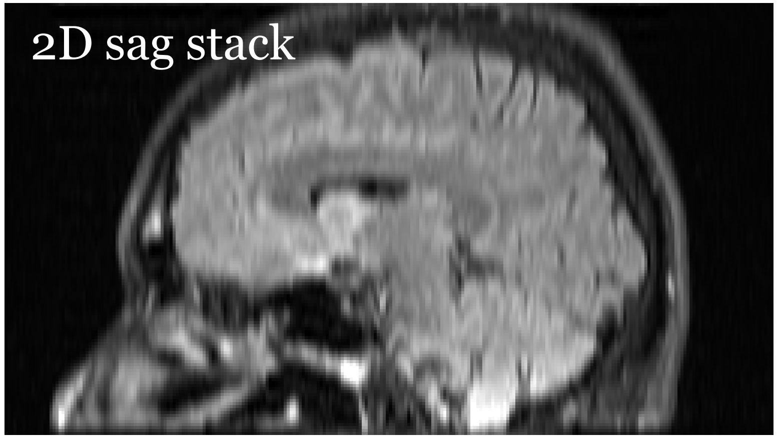
# Examples

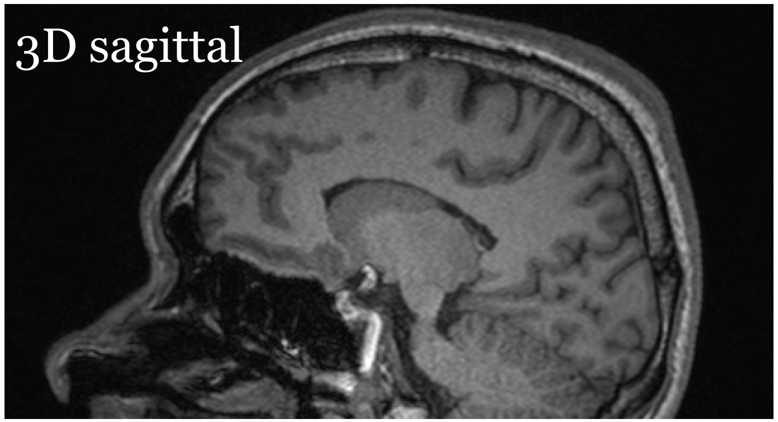


2D k-space



# Homework

- Probs. 10.1 - 10.4, 10.7

# Next Session

Chapter 11 and 12.1-12.2